Math 72 5.3 Factoring by GCF and Grouping
& 5.4 Factoring trinomials.

Objectives

5.3
1) Factor a polynomial using greatest common factor
2) Factor a polynomial by grouping.
3) Solve a polynomial equation using the
   - Zero-product property
   - graph by x-intercept method ("zero")

5.4
4) Factor trinomials with unitary leading coefficient
   x^2+bx+c
   - magic X
   - guess & check
   - boxes
5) Factor trinomials with non-unitary leading coefficient
   ax^2+bx+c $a \neq 0$.
   - magic X
   - guess & check
   - boxes

Factor

(1) $10x^2 + 10x$

both terms are divisible by 10
both terms have x
GCF is 10X

$$= \underset{GCF}{10x}\left(\frac{10x^2}{10x} + \frac{10x}{10x}\right)$$

divide each term by GCF (eventually do this in your head

$$= \boxed{10x(x+1)}$$

check by distribute: $10x \cdot x + 10x \cdot 1$
$= 10x^2 + 10x$ ✓

(2) $12x^3 - 2x^2$

both terms are divisible by 2
both terms have x^2
GCF is $2x^2$

$$= \underset{GCF}{2x^2}\left(\frac{12x^3}{2x^2} - \frac{2x^2}{2x^2}\right)$$

divide each term by GCF

$$= \boxed{2x^2(6x-1)}$$

check by distribute $2x^2 \cdot 6x + 2x^2(-1)$
$= 12x^3 - 2x^2$ ✓

(3) $4z^3 - 12z^2 + 16z$

all 3 terms are divisible by 4
all terms have z
GCF is 4z

$$= 4z\left(\frac{4z^3}{4z} - \frac{12z^2}{4z} + \frac{16z}{4z}\right)$$

$$= \boxed{4z(z^2 - 3z + 4)}$$

check by distribute: $4z(z^2) + 4z(-3z) + 4z(4)$
$= 4z^3 - 12z^2 + 16z$ ✓

(4) $5x^2y + x^3y^2$

coefficients have only GCF 1.
both terms have x^2
both terms have y
GCF = x^2y

$$= x^2y\left(\frac{5x^2y}{x^2y} + \frac{x^3y^2}{x^2y}\right)$$

$$= \boxed{x^2y(5 + xy)}$$

check by distribute: $x^2y \cdot 5 + x^2y \cdot xy = 5x^2y + x^3y^2$ ✓

Factor.

(5) $18x^6 + 3x^3 - 6x^2$

18, 3 and 6 are all divisible by 3
x^6, x^3, x^2 all have x^2
GCF is $3x^2$

$$3x^2\left(\frac{18x^6}{3x^2} + \frac{3x^3}{3x^2} - \frac{6x^2}{3x^2}\right)$$

$$= \boxed{3x^2(6x^4 + x - 2)}$$

check by distribute $3x^2 \cdot 6x^4 + 3x^2(x) + 3x^2(-2)$
$= 18x^6 + 3x^3 - 6x^2$ ✓

(6) $8mn^3 - 2m^2n + 4n$

8, 2 and 4 are all divisible by 2
m, m^2 and no m $\Rightarrow$ no m
n^3, n, n all have n.
GCF $= 2n$

$$= 2n\left(\frac{8mn^3}{2n} - \frac{2m^2n}{2n} + \frac{4n}{2n}\right)$$

$$= \boxed{2n(4mn^2 - m^2 + 2)}$$

check by distribute $2n \cdot 4mn^2 + 2n(-m^2) + 2n \cdot 2$
$= 8mn^3 - 2nm^2 + 4n$ ✓

(7) $-10x^3 + 5x^2 - 15x$

It will be useful to have a positive leading coefficient inside parentheses when we are done, so we will have a (-1) in our GCF because of the first term alone.

10, 5, 15 are all divisible by 5
x^3, x^2 and x all have x

GCF $-5x$

$$= -5x\left(\frac{-10x^3}{-5x} + \frac{5x^2}{-5x} - \frac{15x}{-5x}\right)$$

$$= \boxed{-5x(2x^2 - x + 3)}$$

(+) ☺

check by distribute
$= -5x(2x^2) + (-5x)(-x) + (-5x)(3)$

$= -10x^3 + 5x^2 - 15x$ ✓

ZERO-PRODUCT PROPERTY
If $a \cdot b = 0$ Then either $a = 0$ or $b = 0$ or both.

★ only works if $= 0$ ★ only works if multiplied (product)

Solve the equation using the zero-product property.
Confirm by substituting and also by graphing.

(8) $x(x+1) = 0$

$$\underbrace{x}_{\substack{\text{1st} \\ \# \\ \text{expression}}} \cdot \underbrace{(x+1)}_{\substack{\text{2nd} \\ \# \\ \text{expression}}} = 0$$

<u>step 1</u>: Is it equal to zero?
<u>step 2</u>: Is it a product?

<u>step 3</u>: Set each factor = 0
<u>step 4</u>: Isolate x.

$\boxed{x=0}$ $\quad x+1=0$

$\boxed{x=-1}$

$\boxed{Y=}$ $Y_1 = x(x+1)$ x-intercepts

check by substituting:

$x=0$: $0(0+1) \stackrel{?}{=} 0$

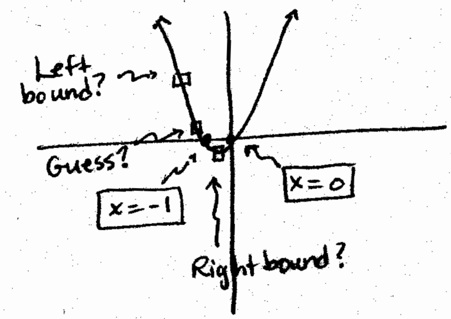
$0(1) = 0$

$0 = 0$ ✓

$x=-1$ $\quad -1(-1+1) \stackrel{?}{=} 0$

$-1(0) = 0$

$0 = 0$ ✓



$\boxed{\text{WINDOW}}$
$[-5, 5, 1] \times [-5, 5, 1]$

(9) $3x(x+2) = 0$

$3x = 0$ $\quad x+2=0$

div by 3 $\quad$ subtract 2

$\frac{3x}{3} = \frac{0}{3}$ $\quad \boxed{x=-2}$

$\boxed{x=0}$

To calculate x-intercepts
$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ (CALC)
2. zero
Left bound? (see graph ↑) $\boxed{\text{enter}}$
Right bound? (" ") $\boxed{\text{enter}}$
Guess? (" ") $\boxed{\text{enter}}$

$\boxed{Y=}$ $Y_1 = 3x(x+2)$
x-intercepts

check by substituting:

$x=0$: $\quad 3(0)(0+2) \stackrel{?}{=} 0$

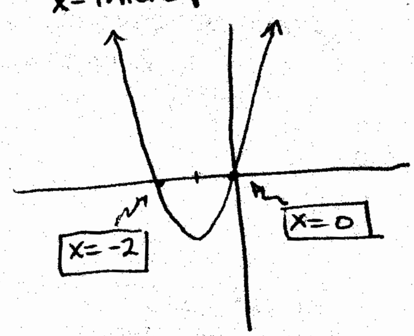
$3(0)(2) = 0$

$0 = 0$ ✓

$x=-2$ $\quad 3(-2)(-2+2) \stackrel{?}{=} 0$

$3(-2)(0) = 0$

$0 = 0$ ✓



⑩ $(3x-2)(2x+1)=0$

check by graphing

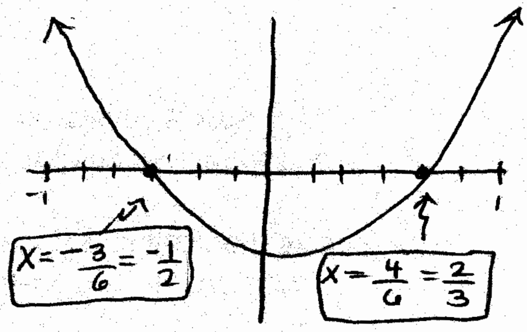
$\boxed{Y=}$ $y_1=(3x-2)(2x+1)$

WINDOW $[-1, 1, \frac{1}{6}] \times [-5, 5, 1]$

$3x-2=0 \qquad 2x+1=0$

$3x=2 \qquad 2x=-1$

$\boxed{x=\frac{2}{3}} = \frac{4}{6} \qquad \boxed{x=-\frac{1}{2}} = -\frac{3}{6}$



check by substituting:

$x=\frac{2}{3}$: $\left(3\cdot\frac{2}{3}-2\right)\left(2\cdot\frac{2}{3}+1\right) \stackrel{?}{=} 0$

$(2-2)\left(\frac{4}{3}+1\right)=0$

$0\left(\frac{7}{3}\right)=0$ ✓

$x=-\frac{1}{2}$ $\left(3\cdot\frac{-1}{2}-2\right)\left(2\cdot\frac{-1}{2}+1\right) \stackrel{?}{=} 0$

$\left(\frac{-3}{2}-2\right)(-1+1)=0$

$\left(\frac{-7}{2}\right)(0)=0$

$0=0$ ✓

⑪ $x^2+5x=0$

step 1: It is equal to zero.

step 2: Is it a product? No. Factor out GCF.

$x(x+5)=0$

$\boxed{x=0} \qquad x+5=0$

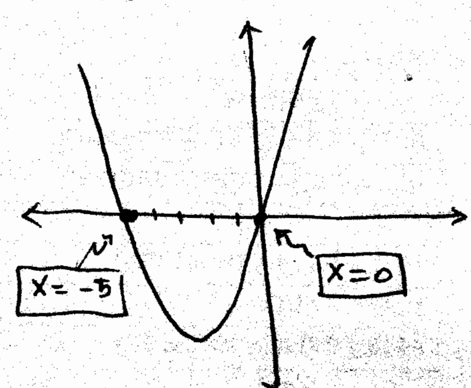
$\boxed{x=-5}$

check by graphing

$\boxed{Y=}$

$y_1=x^2+5x$

STANDARD WINDOW



check by substituting

$x=0 \qquad 0^2+5(0) \stackrel{?}{=} 0$

$0+0=0$ ✓

$x=-5 \qquad (-5)^2+5(-5) \stackrel{?}{=} 0$

$25-25=0$ ✓

(12) $3x^2 = 9x$

$3x^2 - 9x = 0$

$3x(x-3) = 0$

$3x = 0$ $\quad$ $x - 3 = 0$

$\boxed{x=0}$ $\quad$ $\boxed{x=3}$

check by substituting:

$x=0$: $3(0)^2 = 9(0)$

$0 = 0.$ ✓

$x=3$: $3(3)^2 = 9(3)$

$27 = 27$ ✓

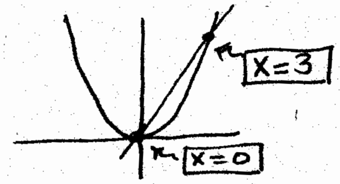
step 1: Is it equal to 0?
no. Subtract 9x both sides

step 2: Is it a product?
no. Factor out GCF

check by graphing, intersection

$\boxed{Y=}$ $Y_1 = 3x^2$
$Y_2 = 9x$

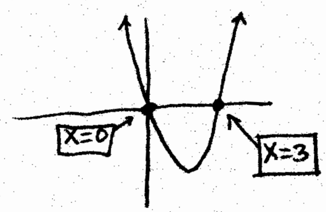
$\boxed{\text{WINDOW}}$ $[-5,5,1] \times [-5,30,5]$



check by graphing, x-intercepts

$\boxed{Y=}$ $Y_1 = 3x^2 - 9x$
Y_2 $\boxed{\text{CLEAR}}$

STANDARD WINDOW



(13) $4x^3 = -4x^2$

$4x^3 + 4x^2 = 0$

$4x^2(x+1) = 0$

$4 \cdot x \cdot x \cdot (x+1) = 0$

$x=0$ $\quad$ $\boxed{x=0}$ $\quad$ $\boxed{x=-1}$

check by subst

$x=0$ $\quad$ $4(0)^3 = -4(0)^2$

$0 = 0$ ✓

$x=-1$ $\quad$ $4(-1)^3 = -4(-1)^2$

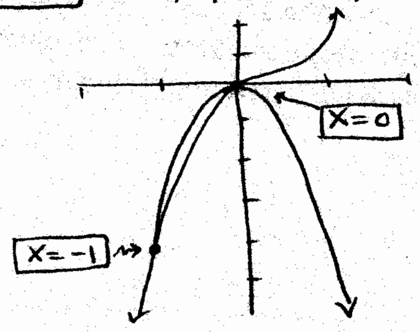
$4(-1) = -4(1)$

$-4 = -4$ ✓

check by graphing, intersection

$\boxed{Y=}$ $Y_1 = 4x^3$
$Y_2 = -4x^2$

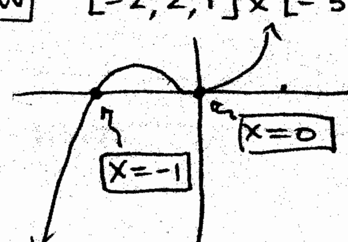
$\boxed{\text{WINDOW}}$ $[-2,2,1] \times [-5,2,1]$



check by graphing x-intercepts

$\boxed{Y=}$ $Y_1 = 4x^3 + 4x^2$
$Y_2 = \boxed{\text{CLEAR}}$

$\boxed{\text{WINDOW}}$ $[-2,2,1] \times [-5,2,1]$



Factor out the binomial (2 terms) GCF

(14) $\underbrace{2x(x+1)}_{\text{1st term}} + \underbrace{3(x+1)}_{\text{2nd term}}$

use add and subtract signs which are outside ( ) to separate terms

factor $(x+1)$ to front

$(x+1)\left[2x\,\frac{\cancel{(x+1)}}{\cancel{(x+1)}} + 3\,\frac{\cancel{(x+1)}}{\cancel{(x+1)}}\right]$

$= \boxed{(x+1)(2x+3)}$

check by FOIL
$2x^2+3x+2x+3$
$2x^2+5x+3$ ✓

distribute original
$2x^2+2x+3x+3$
$2x^2+5x+3$ ✓

Factor.

(15) $\underbrace{3x^2(2x-3)}_{} + \underbrace{x(2x-3)}_{}$

1st term has $x(2x-3)$

2nd term has $x(2x-3)$

Factor $x(2x-3)$ to front.

$=x(2x-3)\left[\frac{3x^2\cancel{(2x-3)}}{\cancel{x(2x-3)}} + \frac{\cancel{x(2x-3)}}{\cancel{x(2x-3)}}\right]$

$=\boxed{x(2x-3)(3x+1)}$

check by distribute original
$=6x^3-9x^2+2x^2-3x$
$=6x^3-7x^2-3x$

check by FOIL and distribute
$=x(6x^2+2x-9x-3)$
$=x(6x^2-7x-3)$
$=6x^3-7x^2-3x$ ✓

Factor by grouping.

(16) $\underbrace{x^3-3x^2}_{\text{group \#1}} + \underbrace{4x-12}_{\text{group \#2}}$

$+$: sign between groups

step 1: Group terms

step 2: Factor GCF from 1st two terms.

Factor a different GCF, using the sign between the groups, from the 2nd two terms

$= x^2(x-3) + 4(x-3)$

x^2: GCF #1; $(x-3)$: GCF #3; $+$: sign between groups; 4: GCF #2; $(x-3)$: GCF #3

step 3: Factor the binomial GCF from each.

If there is no binomial GCF
- check arithmetic
- check signs
- regroup.

$=\boxed{(x-3)(x^2+4)}$

GCF #3

check by FOIL
$x^3+4x-3x^2-12$
$x^3-3x^2+4x-12$ ✓

(17) $\underbrace{12x^3 - 20x^2}_{\substack{\text{GCF \#1} \\ 4x^2}} - \underbrace{6x + 10}_{\substack{\text{GCF \#2} \\ -2}}$

sign between groups goes with GCF #2.

$= 4x^2(\underbrace{3x - 5}_{\text{GCF \#3}}) - 2(\underbrace{3x - 5}_{\text{GCF \#3}})$ factor out -1 makes this $(-)$.

$= \underbrace{(3x-5)}_{\text{GCF\#3}}(4x^2 - 2)$

still has GCF 2!

$= \boxed{2(3x-5)(2x^2-1)}$

check by FOIL & distribute

$= 2(6x^3 - 3x - 10x^2 + 5)$

$= 12x^3 - 6x - 20x^2 + 10$

$= 12x^3 - 20x^2 - 6x + 10$ ✓

(18) $\underbrace{3x - 3y}_{\substack{\text{GCF \#1} \\ 3}} + \underbrace{ax - ay}_{\substack{\text{GCF \#2} \\ a}}$

sign between is +

$= 3(\underbrace{x-y}_{\text{GCF\#3}}) + a(\underbrace{x-y}_{\text{GCF\#3}})$

$= \boxed{(x-y)(3+a)}$

check by FOIL

$= 3x + ax - 3y - ay$

$= 3x - 3y + ax - ay$ ✓

5.4 Trinomials

(19) Multiply

a) $(x+3)(x+4)$

$= x^2 + 4x + 3x + 12$

$= \boxed{x^2 + 7x + 12}$

b) $(x-3)(x-4)$

$= x^2 - 4x - 3x + 12$

$= \boxed{x^2 - 7x + 12}$

c) $(x-3)(x+4)$

$= x^2 + 4x - 3x - 12$

$= \boxed{x^2 + x - 12}$

d) $(x+3)(x-4)$

$= x^2 - 4x + 3x - 12$

$= \boxed{x^2 - x - 12}$

When factoring degree 2 trinomials, those with unitary (1) leading coefficients are easier than those with non-unitary leading coefficients, so we'll start with those.

$$x^2 + bx + c = (x \quad \underbrace{1st)(x \quad 2nd}_{\text{mult (L of FOIL) must be } c})$$

step 1 So we ask: what number combinations multiply to c?

multiply to 12

3 | 4

7 add to

⑳ ex $x^2 + 7x + 12$

$c = 12$

what number combinations multiply to 12?

1×12	-1×-12
2×6	-2×-6
3×4	-3×-4
4×3	-4×-3
6×2	-6×-2
12×1	-12×-1

These are all repeats.

To get a positive result either $(+)(+) = (+)$ or $(-)(-) = (+)$

$$x^2 + bx + c = (x \quad 1st)(x \quad 2nd)$$

I of FOIL

O of FOIL

$$(1st)x + (2nd)x = bx$$

$$1st + 2nd = b$$

step 2: So we ask ... of the number combinations we found in step 1, which ones add to b?

$1 + 12 = 13$ no   no $-1 + (-12) = -13$

$2 + 6 = 8$ no   no $-2 + (-6) = -8$

$3 + 4 = 7$ yes   no $-3 + (-4) = -7$

step 3: write factors

our factors are $\boxed{(x+3)(x+4)}$ or $\boxed{(x+4)(x+3)}$

㉑ Factor $x^2 - 7x + 12$

multiply to 12

−4 | −3

−7 add to

step 1: combinations that multiply to +12

	sum
1×12	$\to 13$
2×6	$\to 8$
3×4	$\to 7$
-1×-12	$\to -13$
-2×-6	$\to -8$
-3×-4	$\to -7$

step 2: Add each combination until you get $b = -7$

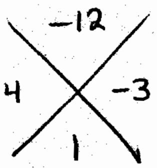
step 3: $\boxed{(x-3)(x-4)}$ OR $\boxed{(x-4)(x-3)}$

Notice: If we change the sign of b, we change the signs on both factors.

(22) Factor x^2+x-12

Step 1: combinations that multiply to -12

To get a negative result, we need one negative and one positive. $(+)(-)=(-)$ or $(-)(+)=(-)$



-1×12	$-1+12=11$	no
-2×6	$-2+6=4$	no
-3×4	$-3+4=1$	yes
-4×3	$-4+3=-1$	no
-6×2	$-6+2=-4$	no
-12×1	$-12+1=-11$	no

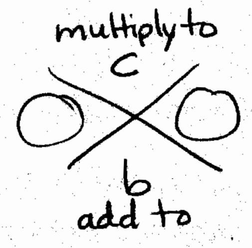
Step 2: add each combination

Step 3: write the factors.

$\boxed{(x-3)(x+4)}$

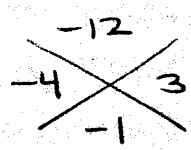
One way to organize steps 1 and 2 is to write a "magic X".

To factor x^2+bx+c



resulting numbers go on the sides.

(23) Factor x^2-x-12



-1×12
-2×6
-3×4
-4×3
-6×2
-12×1

$=\boxed{(x-4)(x+3)}$

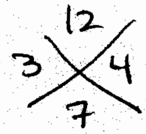
Notice: If we change the sign of b, we change the signs on both factors.

But what if the leading coefficient is not 1?
First, try to factor out a GCF!

(24) $4x^2 + 28x + 48$

$= 4(x^2 + 7x + 12)$

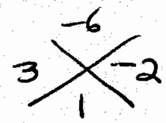
$= \boxed{4(x+3)(x+4)}$



(25) $3x^3 + 3x^2 - 18x$

$= 3x(x^2 + x - 6)$

$= \boxed{3x(x+3)(x-2)}$



But what if it's not a GCF?

(26) Multiply $(5y-2)(2y+1)$

$= 10y^2 + 5y - 4y - 2$

$= \boxed{10y^2 + y - 2}$

(27) Factor $10y^2 - y - 2$

structure $(\ y\quad)(\ y\quad)$

These #'s must multiply to -2

1×-2

-1×2

These numbers must multiply to 10

1×10

2×5

5×2

10×1

Method 1: Guess and check. Put together all the possible combinations, and FOIL to see which one works.

$\left.\begin{array}{l} \cancel{(y+1)(10y-2)} \\ \cancel{(y-1)(10y+2)} \end{array}\right\}$ these will have the same b with opposite signs

$\left.\begin{array}{l} (y+2)(10y-1) \\ (y-2)(10y+1) \end{array}\right\}$

$(2y+1)(5y-2)$
$(2y-1)(5y+2)$
$\cancel{(2y+2)(5y-1)}$
$\cancel{(2y-2)(5y+1)}$

← $(2y+2)$ and $(2y-2)$ both have GCF 2. If there were a GCF of 2, we could have found it at the start.

$\cancel{(5y+1)(2y-2)}$
$\cancel{(5y-1)(2y+2)}$

$(10y+1)(y-2)$
$(10y-1)(y+2)$
$\cancel{(10y+2)(y-1)}$
$\cancel{(10y-2)(y+1)}$

$(10y+2)$ and $(10y-2)$ both have a GCF 2, but the original problem does not have a GCF of 2.

So in the end, we only ~~need~~ need to check:

$(y+2)(10y-1) = 10y^2 - y + 20y - 2 = 10y^2 + 19y - 2$ no.

$(2y+1)(5y-2) = 10y^2 - 4y + 5y - 2 = 10y^2 + y - 2$

⇑ right number, wrong sign.

change both signs in our factors

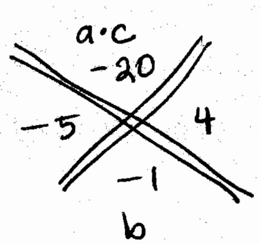
$(10y+1)(y-2)$

answer $\boxed{(2y-1)(5y+2)}$

Method 2: The a·c method, also called rewrite and group, also called magic double X. Can be done with boxes.

$10y^2 - y - 2$

$a=10$ $b=-1$ $c=-2$



Step 1:

multiply $a \cdot c = 10(-2) = -20$ and write at top of double X.

b at the bottom.

Find two numbers that multiply to ac and add to b.

*CAUTION: WE'RE NOT DONE! *

Step 2: Rewrite the middle term using the numbers found and like terms.

$$10y^2 - y - 2$$

$-1y$ becomes $-5y + 4y$ or $4y - 5y$

$$= 10y^2 - 5y + 4y - 2$$

Step 3: Factor by grouping

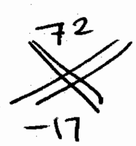
$$\underbrace{10y^2 - 5y}_{\text{GCF \#1}} + \underbrace{4y - 2}_{\text{GCF \#2}}$$

$$= 5y\underbrace{(2y-1)}_{\text{GCF \#3}} + 2\underbrace{(2y-1)}_{\text{GCF \#3}}$$

$$= \boxed{(2y-1)(5y+2)}$$

(28) $12r^2 - 17r + 6$

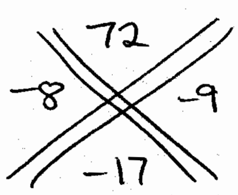
12 is not a GCF. Bummer.



$$\begin{array}{r} 12 \\ \times 6 \\ \hline 72 \end{array}$$

-1×-72
-2×-36
-3×-24
-4×-18
-6×-12
-8×-9

To multiply to (+) but add to (−) need two (−) numbers



$$= \underbrace{12r^2 - 8r}\underbrace{-9r + 6}$$

$$= 4r(3r-2) - 3(3r-2)$$

$$= \boxed{(3r-2)(4r-3)}$$